Registration No.:

Total Number of Pages:02   **Course: IDD/B.Tech**
                                                                   Sub_Code: HSBS2001

3rd Semester Regular/Back Examination: 2025-26
SUBJECT: Mathematics-III

BRANCH(S): E, AEIE, AERO, AI, AME, AUTO, BIOMED, BIOTECH, C&EE, CHEM, CIVIL, CSE, CSEAI, CSEAIML, CSEDS, CSEIOT, CSIT, CST, ECE, EEE, EEVDT, ELECTRICAL, ELECTRICAL & C.E, ELECTRONICS & C.E, ENV, ETC, MANUTECH, MECH, METTA, MINERAL, MINING, MME, MMEAM, PLASTIC, PT

Time: 3 Hour
Max Marks: 100
Q.Code:U495
Answer Question No.1 (Part-I) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q1   **Answer the following questions:**   **(2 x 10)**
a)   Write the existence conditions of Laplace transform method.
b)   Determine the Laplace transform of 9^t.
c)   Write the one dimensional wave equation and one dimensional heat equation.
d)   Define Fourier series for the function having period 2π.
e)   Find the mean of the random variable X with the probability function $f(x) = 2x$.
f)   Define moment of the random variables
g)   Find the probability of getting exactly 2 heads in 6 tosses of a fair coin.
h)   Explain Normal distribution with an example.
i)   Define random sample of size n.
j)   Write two properties of correlation coefficients.

Part-II

Q2   **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)**   **(6 x 8)**
a)   State and prove second shifting theorem.
b)   Use convolution theorem to find $L^{-1}\left\{\frac{1}{(s^2-5s+6)}\right\}$.
c)   Solve the differential equation $y'' + 5y' + 6y = \delta(t)$.
d)   Find the Fourier series of the function $f(x) = -1(-1 < x < 0), \; f(x) = 1(0 < x < 1)$ P=2L=2.
e)   Solve the boundary value problem $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, \; u(0,t) = u(3,t) = 0$ and $u(x,0) = 5\sin 4\pi x - 3\sin 8\pi x$.
f)   Show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots\ldots\ldots\ldots. = \frac{\pi}{4}$

g) Two ball point pensare selected at random from a box that contains 3 blue pens, 2 red pens and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected, find the joint probability function $f(x, y)$.

h) Is continuous random variable X, having probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & otherwise \end{cases} ? \quad \text{Find} \quad P(0 < X \leq 1).$$

i) Find the mean and variance of Normal distribution.

j) The probability that a patient recover from a rare blood disease is 0.4. If 5 people are known to be contaminated the disease, what is the probability that
(i) At least 10 people survive. (ii) Exactly 5 people survive.

k) Establish the relation between sampling distribution and central limit theorem.

l) Explain maximum likelihood estimation of the parameters with a suitable counterexample.

Part-III
Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 (a) Use Laplace transform method to solve the initial value problem: $y'' + y = t^2 + 2t, y(0) = 4, y'(0) = -2$. **(16)**

(b) Solve the integral equation $y = \sin t - \int_0^t y(\tau)\sin(t-\tau)d\tau$.

Q4 a) Find the solution of the heat conduction problem $\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2}, 0 < x < e, \ t > 0,$ **(16)**
$u(0, t) = 0, \forall \ t \geq 0, u(e, t) = 0, \forall \ t \geq 0, u(x, 0) = f(x), 0 \leq x \leq e.$

(b) Define probability distribution function for discrete random variables. Find the probability distribution of boys and girls in families with 3 children, assuming equal probabilities for boys and girls.

Q5 (a) Show that the mean and variance of the binomial distribution $b(x; n, p)$ are $\mu = np \ and \ \rho^2 = npq$. **(16)**

(b) Let X and Y be two independent random variables. Then show that $E(XY) = E(X)E(Y)$.

Q6 (a) The grade of a class 9 students on a midterm report (x) and the final examination (y) as follows **(16)**

x	77	50	71	72	81	94
y	82	66	78	34	47	85

Estimate the linear regression line.

(b) Compute and interpret the correlation coefficient for the following data of 6 students selected at random.

Mathematics grade	70	92	80	74	65	83
English grade	74	84	63	87	78	90