Registration No.:

Total Number of Pages: 02

Course: B.Tech/IDD (B.Tech and M.Tech)
Sub_Code: RMA2A001

2nd Semester Back Examination: 2024-25
SUBJECT: Mathematics-II
BRANCH(S): AE, AEIE, AUTO, CHEM, CIVIL, CSE, CSEAI, CSEAIML, CSEDS, CSIT, CST, ECE, EEE, ELECTRICAL, ELECTRICAL & C.E, ELECTRONICS & C.E, ETC, IT, MECH, METTA, MINERAL, MINING, MME, PLASTIC
Time: 3 Hours
Max Marks: 100
Q.Code : S260
Answer Question No.1 (Part-1) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right-hand margin indicate marks.

Part-I

Q1 Answer the following questions: **(2 x 10)**

- **a)** If the rank of the matrix A is m and the rank of the matrix B is n. Then what can you say about the rank of the matrix AB?
- **b)** What type of matrix is obtained after applying the Gauss Elimination Method?
- **c)** If A is a Hermitian matrix, then show that iA is skew-Hermitian.
- **d)** Is the matrix $A = \begin{bmatrix} 3 & 5 & 7 \\ 5 & 8 & 11 \\ 7 & 11 & 13 \end{bmatrix}$ diagonalizable? Justify your answer.
- **e)** Find the divergence of $\vec{F}(x,y,z) = xyz\hat{i} + x^2y^2z^2\hat{j} + y^2z^3\hat{k}$.
- **f)** Find the arc length of $y = \ln(\cos x); x = 0 \ to \ x = \frac{\pi}{3}$.
- **g)** Use Green's theorem to evaluate $\int_C (y^2dx + x^2dy)$, where C is the square with vertices $(0,0),(1,0),(1,1) \ and \ (0,1)$
- **h)** State Stokes theorem.
- **i)** Define even function. Give an example of it.
- **j)** Find the Fourier integral representation of $f(t) = e^{-k|t|}, k > 0$.

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) **(6 x 8)**

- **a)** Show that every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew-symmetric matrix
- **b)** Find the inverse of the matrix by using Gauss-Jordan method $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
- **c)** Solve the following system of linear equations
  $2x_1 + 2x_2 + x_3 = 9,\ x_1 + 3x_2 + 2x_3 = 10, 2x_1 - x_2 + 2x_3 = 4$

d) Find the eigenvalue and eigenvecotors of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

e) Define unitary matrix. Discuss about the eigenvalues of it.

f) Prove that $\overline{A}$ is Hermitian and skew-Hermitian according as A is Hermitian or skew-Hermitian.

g) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1,2,-1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

h) Apply Green's theorem to evaluate $\oint_C (2x^2 - y^2)dx + (x^2 + y^2)dy$, where C is the boundary of the surface in the xy-plane enclosed by the x-axis and the semi-circle $y = \sqrt{4 - x^2}$.

i) Apply Gauss divergence theorem to evaluate $\oiint_S (ax\hat{i} + by\hat{j} + cz\hat{k}).\hat{n}ds$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

j) Find the Fourier series expansion valid in $(0, 2)$ of the function $f(x) = 4 - x^2$.

k) Find the Fourier sine transform of $e^{-at}, a > 0$.

l) Find the Fourier transform of $f(t) = \frac{1}{t^4 + 1}$.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four) **(16 x 2)**

Q3 **a)** Define vector space. Give an example of a vector space. **(8 x 2)**

b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$.

Q4 **a)** If possible digonalize the matrix $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ 1 & 5 & 6 \end{bmatrix}$. **(8 x 2)**

b) Show that all the eigenvalues of an Hermetian matrix are always real numbers.

Q5 Define the arc length of a curve. Derive the formula for arc length in Cartesian form. Prove that the length of the loop of the curve $3ay^2 = x(x-a)^2$ is $\frac{4a}{\sqrt{3}}$. **(16)**

Q6 Find the whole-range Fourier series expansion for $f(x)$ for $0 < x < 2\pi$, where **(16)**

$$f(x) = \begin{cases} (x-\pi)^2, & for\ 0 < x < \pi \\ \pi^2, & for\ \pi < x < 2\pi \end{cases}$$

Hence, deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.