Total Number of Pages: 02

Course: B.Tech
Sub_Code: RMA1A001

**1st Semester Back Examination: 2025-26**
**SUBJECT: MATHEMATICS-I**
**BRANCH(S): AE, AG, AUTO, CHEM, CIVIL, CSE, CSEAI, CSEAIML, CSEDS, CSIT, CST, ECE, EEE, ELECTRICAL, ELECTRICAL & C.E, ELECTRONICS & C.E, ETC, IT, MANUTECH, MECH, MINING, MME, PLASTIC**
**Time: 3 Hours**
**Max Marks: 100**
**Q.Code: U508**
**Answer Question No.1 (Part-I) which is compulsory, any eight from Part-II and any two from Part-III. The figures in the right-hand margin indicate marks.**

### Part-I

| | | |
|---|---|---|
| **Q1** | **Answer the following questions:** | **(2 x 10)** |
| **a)** | Explain parallel asymptotes with examples. | |
| **b)** | Write the necessary condition for a function $f(x,y)$ to have an extremum. | |
| **c)** | Prove or disprove that the integrating factor for the non-exact ordinary differential equations (ODEs) is unique. | |
| **d)** | Solve the ODE $xdy - ydx = 0$. | |
| **e)** | Compute the value of the expression $\frac{1}{D}(logx)$. | |
| **f)** | Write the general form of Euler-Cauchy equation. | |
| **g)** | Write the power series of the function $f(x) = e^{-x^2}$. | |
| **h)** | Write the Bessel's equation of the first kind. | |
| **i)** | Find the Laplace transform of $\cosh at$. | |
| **j)** | Determine the inverse Laplace transform of 1. | |

### Part-II

| | | |
|---|---|---|
| **Q2** | **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** | **(6 x 8)** |
| **a)** | Find the asymptotes to the curve $x^3 + y^3 - 3axy = 0$. | |
| **b)** | Establish the relation between Beta function and Gamma function. | |
| **c)** | Solve the ODE $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$. | |
| **d)** | Check whether the ODE $(x^2 + y^2 + 2x)dx + 2ydy = 0$ is exact. Also solve it. | |
| **e)** | Find the general solution of the second order homogeneous ODE $4y'' + 4y' - 3y = 0$. | |
| **f)** | Find the general solution of the ODE $y''' + 4y'' + 4y = 8e^{-2x}$. | |
| **g)** | Solve the ODE $y'' + y = 0$ using power series method | |
| **h)** | Solve the ODE $y'' + y = 10e^{2x}$ using the method of undetermined coefficients. | |
| **i)** | Show that the Bessel's function of first kind satisfies the recurrence relation $J_{v+1} - J_{v-1} = -2J_v'$. | |
| **j)** | Find the Convolution $\sin \omega t * \cos \omega t$. | |

**k)** Solve the integral equation $y = \sin t - \int_0^t y(\tau)\sin(t-\tau)d\tau$.

**l)** Compute $L^{-1}\left\{\frac{3s-2}{s^2-4s+20}\right\}$.

## Part-III
### Only Long Answer Type Questions (Answer Any Two out of Four)

**Q3** **a)** Find the radius of curvature at any point of the curve $(r, \theta)$ for the curve $r = a(1 - cos\theta)$. **(8 x 2)**

**b)** Find the relative extrema and saddle point of the function $f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5 = 0$.

**Q4** Solve the ODEs (I) $\frac{dy}{dx} + 2xy = e^{-x^2}$ (II) $x\frac{dy}{dx} + y = xy^2$. **(8 x 2)**

**Q5** **a)** Find the power series solution of the of the ordinary differential equations $y'' + y' - y = 0$ about the point $x = 2$. **(8 x 2)**

**b)** Show that $(2n + 1)xP_n(x) = (n + 1)P_{n+1}(x) + n P_{n-1}(x)$.

**Q6** **a)** Use Laplace transform method to solve the initial value problem $y'' - 3y' + 2y = e^{2t}, y(0) = 3, \ y'(0) = 5$. **(8 x 2)**

**b)** Solve the ODE $y'' + 2y' + y = e^{-x}\log x$ using the method of variation of parameters.