Registration No.: | | | | | | | | | | |

Total Number of Pages: 02 **Course: B.Tech/IDD**
Sub_Code: 23BS1001

1st Semester Regular/Back Examination: 2025-26
SUBJECT: MATHEMATICS-I
BRANCH(S): AE, AEIE, AERO, AG, AI, AIML, AME, AUTO, BIOMED, BIOTECH, CHEM, CIVIL, CS, CSE, CSE(CS), CSEAI, CSEAIML, CSEDS, CSEIOT, CSIT, CST, CST, ECE, EEE, EEVDT, ELECTRICAL, ELECTRICAL & C.E, ELECTRONICS & C.E, ENV, ETC, IT, MANUTECH, MECH, METTA, MINERAL, MINING, MME, MMEAM, PLASTIC
Time: 3 Hours
Max Marks: 100
Q.Code: U507
Answer Q1 (Part-I) which is compulsory, any eight from Part-II, and any two from Part-III.
The figures in the right-hand margin indicate marks.

Part-I

Q1 **Answer the following questions:** **(2 x 10)**

- **a)** Compute $\Gamma\left(\frac{7}{2}\right)$.
- **b)** Define an Improper Integral with an example.
- **c)** Describe the geometrical interpretation of Rolle's theorem.
- **d)** Write the Maclaurin series expansion for $f(x) = cosx$.
- **e)** If $u = x^2y + y^2x$ find $\frac{\partial^2 u}{\partial x \partial y}$.
- **f)** Define the Hessian Matrix for a function $f(x, y)$.
- **g)** Explain Linear Dependence of a set of vectors with a suitable example.
- **h)** Find the Rank of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 1 \\ 4 & 3 & 7 \end{pmatrix}$.
- **i)** State the Cayley-Hamilton Theorem.
- **j)** If λ is an eigenvalue of a non-singular matrix A, what is the eigenvalue of A^{-1}?

Part-II

Q2 **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6 x 8)**

- **a)** Find the length of the arc of the curve $y = ln(sec\,x)$ from $x = 0\ to\ x = \frac{\pi}{3}$.
- **b)** Find the volume of the solid generated by revolving the region bounded by $y = x^2$ and $y = 4$ about the y-axis.
- **c)** Show that $\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.
- **d)** Verify the Mean Value Theorem for $f(x) = x^3 - 5x^2 - 3x$ in the interval $[1,3]$.

e) Use the First Derivative Test to find the local extrema of $f(x) = x^3 - 6x^2 + 9x + 15$.

f) Using Taylor's series, derive the Maclaurin expansion of $\ln(1 - x)$.

g) If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin\,2u$.

h) Find the Jacobian $\frac{\partial(u,v)}{\partial(n,y)}$ if $u = x(1 - y)$ and $v = xy$.

i) Expand the set of vectors $\{(1,1,0)\}$ to a basis of $\mathbb{R}^3$.

j) Solve the system of equations $2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16$ using Gauss Elimination method.

k) Find the inverse of the matrix $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$ using the Gauss-Jordan Method.

l) Show that the matrix $A = \frac{1}{3}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix}$ is orthogonal.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 **a)** Find the area of the surface formed by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-$axis$. **(8 x 2)**

b) Define Mean value theorem. Prove that between any two real roots of $e^x\,sin\,x = 1$, there is at least one real root of $e^x\,cos\,x + 1 = 0$.

Q4 **a)** Find the maximum and minimum values of the function $f(x,y) = x^3 + y^3 - 3xy$. Determine the nature of the saddle point if it exists. **(8 x 2)**

b) Write the Lagrange's multipliers method. Find the extreme values of the function $f(x,y) = xy$ subject to $2x + 2y = 5$.

Q5 **a)** Find the values of $\lambda\ and\ \mu$, so that the system of equations $x + y + z = 6, x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ has (i) no solution, (ii) a unique solution, (iii) infinite solutions. **(8 x 2)**

b) Find the basis and dimension of the subspace spanned by the vectors $(1,2,3), (2,3,4), (3,5,7)$.

Q6 **a)** Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$. **(8 x 2)**

b) Verify the Cayley-Hamilton Theorem for the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and use it to find A^{-1}.