Registration No.: | | | | | | | | | | |

Total Number of Pages: 02 **Course: B.Tech**
Sub_Code: CSBS2001

3rd Semester Regular/Back Examination: 2025-26
SUBJECT: DISCRETE MATHEMATICS
BRANCH(S): CSE, CSEAI, CSEAIML, CSEDS, CSIT, CST, IT
Time: 3 Hours
Max Marks: 100
Q.Code: U494

Answer Q1 (Part-I) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right-hand margin indicate marks.

Part-I

Q1 **Answer the following questions:** **(2 x 10)**

a) Construct the truth table for the implication $p \rightarrow q$.
b) Suppose $A = \{1, 2, 3\}$. Construct the Power Set $P(A)$ and state its cardinality.
c) State the Pigeonhole Principle.
d) Explain injective and surjective functions with examples.
e) Compute the first four terms of the sequence defined by $a_n = 2a_{n-1}\ and\ a_0 = 1$.
f) Define an Equivalence Relation. Provide an example of it.
g) If G is a group such that $(a \cdot b)^2 = a^2 \cdot b^2, \forall a, b \in G$, then Show that G is abelian.
h) Give an example of an abelian group which has exactly 4 elements.
i) Explain graph isomorphism with a suitable example
j) A graph has 12 edges, two vertices of degree 4, and other vertices of degree 5. Find the number of vertices in the graph.

Part-II

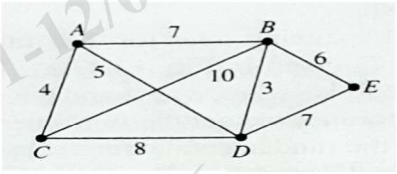
Q2 **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6 x 8)**

a) Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held" and "The sailing race is not held" imply the conclusion "It rained".
b) Let $A, B, and\ C$ be three sets. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
c) Let $f(x) = 2x + 3\ and\ g(x) = 3x + 2$ be two real valued functions. Compute the composition $(f\ o\ g)(x)$ and $(g\ o\ f)(x)$. Are the compositions equal?
d) In how many ways can a committee of 5 people be chosen from 10 men and 8 women if the committee must contain at least 3 women?
e) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$ and given $a_0 = 1, a_1 = 4$.
f) Let R be a relation on the set of integers Z defined by $a\ R\ b$ if and only if $a \equiv b(mod\ 5)$. Prove that R is an equivalence relation.
g) Prove that the intersection of two subgroups of a group G is also a subgroup of G.

h) Show that Kernel of homomorphism of a group G is a normal subgroup of G.

i) Suppose G is a group of order 45 with a normal subgroup of order 9. Then prove that G must be abelian.

j) Define minimum spanning tree. Use Kruskal's algorithm to find a minimum spanning tree of the following graph.



k) Using Kuratowski's theorem, determine whether the Peterson's graph is plannar or not.

l) Define vertex coloring. Prove that every plannar graph is 6-vertex colorable.

Part-III
Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 **a)** What is a Tautology? Construct the truth table of $(P \rightarrow Q \wedge R) \vee (\neg P \wedge Q)$. **(8 x 2)**

b) Use Mathematical Induction to *compute* the sum of the first n odd integers.

Q4 **a)** Let $A = \{1,2,3,4,6,12\}$ and the relation R be "divides" $(a|b)$. Draw the Hasse Diagram for this partial ordering. Identify the maximal, minimal, greatest, and least elements if they exist. **(8 x 2)**

b) Discuss the closure of relations. Find the reflexive, symmetric, and transitive closures of the relation $R = \{(1,2), (2,3), (3,4)\}$ defined on the set $A = \{1,2,3,4\}$.

Q5 **a)** State and prove Lagrange's Theorem for finite groups. Also discuss about the converse. **(8 x 2)**

b) Define an integral domain Prove or disprove that every integral domain is a field.

Q6 **a)** What is the Traveling Salesperson Problem (TSP)? Explain the difference between finding a Euler Circuit and a Hamilton Circuit. Why is TSP considered computationally hard? **(8 x 2)**

b) For every simple plannar graph G with n –vertices and e –edges, prove that $e \leq 3n - 6$.