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Total Number of Pages : 02

Course: B.Tech  
Sub\_Code: PMA4E001

4<sup>th</sup> Semester Back Examination: 2022-23

SUBJECT: Applied Mathematics-III

AEIE,AUTO,CIVIL,CSE,EEE,ELECTRICAL,ECE,ETC,EIE,IT,MECH,METTA,MINING,PE

Time : 3 Hour

Max Marks : 100

Q.Code : M214

Answer Question No.1 (Part-I) which is compulsory, any eight from Part-II and any two from Part-III.

The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions:

(2 x 10)

- Give an example of a function which is differentiable but not analytic.
- Obtain the Taylor series representation of  $\cos z$  at  $z = 0$ .
- State Cauchy-Riemann equations.
- Find the residue of  $\frac{1}{(z^2+1)^2}$  at  $z = i$ .
- Write the disadvantages of Lagrange interpolation.
- Write the error term in interpolations.
- Find  $y(0.2)$  by improved Euler's method, given that  $y' = x + y$ ;  $y(0) = 1$  with  $h = 0.1$
- Find the probability of getting exactly 2 heads in 6 tosses of a fair coin.
- Write a short note on regression analysis.
- Find the mean of the random variable  $X$  with the probability function  $f(x) = 2x$ .

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) (6 x 8)

- Develop  $f(z) = \frac{2z-3i}{z^2-3iz-2}$  in a Laurent series valid for  $1 < |z| < 2$ .
- Find the Maclaurin series expansion of  $\frac{1}{z+3i}$ .
- Evaluate  $\oint_C (z - z_1)^{-1}(z - z_2)^{-1} dz$ .  $C$  is a simple closed path enclosing  $z_1$  and  $z_2$ , which are arbitrary.
- Evaluate  $\int_{|z|=1} \frac{dz}{z^2+4z+1}$  using residue theorem.
- If  $f(x) = \frac{1}{x}$ , then show that  $f[x_0, x_1, \dots, x_n] = \frac{(-1)^n}{x_0 x_1 \dots x_n}$ .
- Use Gaussian three-point formula to evaluate  $I = \int_1^5 \frac{dz}{z}$ .

- g) Determine the ranges for  $\alpha$  and  $\beta$ , show that the below function is cubic spline:

$$f(x) = \begin{cases} \alpha x^3 + \beta x^2 + 2x, & -1 \leq x \leq 0 \\ 3x^3 + x^2 + 2x, & 0 \leq x \leq 1. \end{cases}$$

- h) Determine the interpolating quadratic polynomial for the given data using Newton's forward difference formula

$x$	0	2	4	6
$f(x)$	-3	5	21	45

- i) Find the approximate solution of  $y' + y = 0, y(0) = 1$  using Euler's method, find  $y(0.04)$ . Compare your answer with the exact solution.
- j) Define probability distribution function for discrete random variables. Find the probability distribution of boys and girls in families with 3 children, assuming equal probabilities for boys and girls.
- k) Calculate the correlation coefficient from the following data:

X:	12	9	8	10	11	13	7
Y:	14	8	6	9	11	12	3

- l) What is sampling distribution? Find the sampling distribution of mean from Poisson distribution.

### Part-III

#### Only Long Answer Type Questions (Answer Any Two out of Four)

- Q3** State and prove Cauchy integral theorem. Also discuss its converse part. **(16)**
- Q4** Define singularity. Evaluate the integrals  $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$  using Residue Theorem. **(16)**
- Q5** a) Find the value of  $\log 2$  from  $\int_0^1 \frac{x^2}{1+x^3} dx$  using Simpson's one-third rule with  $h = 0.25$ . **(8x2)**
- b) Using fourth order Runge-Kutta method, find  $f(0.8)$  if  $y' = y - x^2 + 1, y(0) = 0.5, h = 0.2$ .
- Q6** (a) Define uniform distribution and show that the mean and variance of uniform distribution are given by  $\mu = \frac{1}{2}(a + b)$  and  $\sigma^2 = \frac{1}{12}(b - a)^2$ , respectively. **(8x2)**
- (b) The manufacturer of a patent medicine claimed that it was 90% effective in relieving an allergy for a period of 8 hours. In a sample of 200 people who had the allergy, the medicine provided relief for 160 people.
- (i) Determine whether the manufacturer's claim is legitimate by using 0.01 as the level of significance.
- (ii) Find the P value of the test.