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Total Number of Pages : 02

B. Tech& IDD(B.Tech and M.Tech)
RMA2A001

4th Semester Regular/Back Examination: 2023-24

Discrete Mathematics

CSE, CSEAI, CSEAIME, CSEDS, CST, IT

Time : 3 Hour

Max Marks : 100

Q.Code : P057

Answer Question No.1 (Part-1) which is compulsory, any eight from Part-II and any two from Part-III.

The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions : (2 x 10)

- Give an example of ring which is not an integral domain.
- Find all the left cosets of the subgroup $(3\mathbb{Z}, +)$ for the group $(\mathbb{Z}, +)$.
- What is the order of element $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ in the permutation group S_3 under composition?
- Find the distance between 111001 and 011010
- Simplify the Boolean expressions $xyz + \bar{x}y + xyz$
- Is bipartite graph a tree? Draw a 3-regular graph.
- Define (a) complemented and (b) distributive lattice.
- Find the number of edges in a graph G having 3 vertices of degree 4 and 4 vertices of degree 3
- In any group of 27 English words, there must be at least _____ that begins with the same letter.
- Find the converse statement "If you want to conduct an online meeting, then you have to create a meeting link."

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight (6 x 8) out of Twelve)

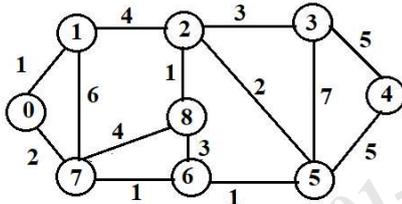
- Using mathematical induction prove that $3 + 3.5 + 3.5^2 + \dots + 3.5^n = \frac{3(5^{n+1}-1)}{4}$ for positive integer n
- Solve the recurrence relation using the generating function $a_n = a_{n-2}$ given that $a_0 = 0$ and $a_1 = 1$.
- If G is a simple connected planar graph with e edges and v vertices where $v \geq 3$, then $e \leq 3v - 6$
- Using the truth table prove that $\sim(p \rightarrow q) \equiv p \wedge \sim q$

- e) Determine the validity of the argument, "Robbery was the motive for the crime only if the victim had money in his pockets. But robbery or revenge was the motive for the crime. Therefore, revenge must have been the motive for the crime"
- f) Is K_4 a Hamilton circuit? Justify
- g) Explain (a) Spanning Tree (b) Boolean Algebra c) Karnaugh map
- h) Verify whether R is equivalence, $L = \{\text{all real lines in a plane}\}$ and $R = \{(l, m) \mid l \text{ is perpendicular to } m\}$.
- i) Without using a truth table prove that $\sim p \wedge (p \vee q) \equiv \sim p \wedge q$
- j) A ring R is commutative if and only if $(a + b)(a - b) = a^2 - b^2, \forall a, b \in R$
- k) Let x, y are real numbers and $Q(x, y): xy = 1$. What is the truth value of (a) $\exists y \forall x Q(x, y)$, (b) $\forall y \forall x Q(x, y)$?
- l) Let the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = \frac{x}{2}$. Is it one-one? Is it onto?

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

- Q3** a) What is the chromatic number of $K_{2,3}$? (2+8+6)
 b) Using Kruskal's algorithm, find the minimum spanning tree



- c) Show that in a ring $(R, +, \cdot)$, cancellation laws hold under addition.

- Q4** Given the poset $(\{1, 2, 3, 5, 6, 7, 10, 20, 30, 60, 70\}, /)$ where $/$ denotes division. (3+3+3+2+3)
 i) Draw the Hasse Diagram for this poset. ii) Find the minimal elements.
 iii) Find the maximal elements. iv) Find the greatest element v) Find the least element vi) Is this poset a lattice? Justify your answer.

- Q5** Let $S = \{a, b, c, d\}$ and $R = \{(a, b), (b, c), (c, d), (b, a)\}$. Find the transitive closure of R using Warshall's algorithm. Define Ring. (12+4)

- Q6** Prove that $G = \{1, 2, 3, 4\}$ form a group under multiplication modulo 5. (8+8)
 Let $f: G \rightarrow H$ is a group homomorphism. Then $f(e_G) = e_H$ and for any $g \in G, f(g^{-1}) = f(g)^{-1}$