Registration No :

Total Number of Pages : 02

Course: B.Tech
Sub_Code: RCS4C001

4th Semester Regular/Back Examination: 2022-23
SUBJECT: Discrete Mathematics
BRANCH(S): CST,CSEAI,CSE,CSEAIME
Time : 3 Hour
Max Marks : 100
Q.Code : M218
Answer Question No.1 (Part-1) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q1 **Answer the following questions:** **(2 x 10)**

a) Show that $\sim \forall x\big(P(x) \rightarrow Q(x)\big)$ and $\exists x\big(P(x) \wedge\sim Q(x)\big)$ are logically equivalent.

b) Let C be "Today is clear", R be "It is raining today" and S be "It is snowing today". Then translate the symbolic notation $C \rightarrow \neg(R \wedge S)$ into acceptable English.

c) Using induction, show that $1 + 2 + 3 + \cdots n = \frac{n(n+1)}{2}$.

d) Define Partial order set. Also provide suitable example of it.

e) Suppose $A = \{1,2,3,4\}$. Which order pair(s) are in the relation $R = \{(a, b) \mid a \text{ divides } b\}$?

f) Solve the recurrence relation $a_n = 6\, a_{n-1} - 9\, a_{n-2}$.

g) Give an example of an abelian group which has exactly 4 elements.

h) Define Lattice.

i) Prove or disprove that a simple digraph is remain a simple graph after removing its direction.

j) How many edges must be there in a planar graph having 7 regions and 5 vertices?

Part-II

Q2 **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6 × 8)**

a) Translate the statement "If either labor or management is stubborn, then the strike will be settled iff the government obtains an injuction, but troops are not sent into the mills. '' in symbolic form. Also construct its truth table.

b) Prove or disprove that every partial order sets are totally ordered.

c) Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with the initial conditions $a_0 = 1, a_1 = 6$.

d) Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer.

e) Students are awarded 4 grades A, B, C, and D. How many students must be there in a group so that at least 6 students get the same grade?

f) Let R be the relation on the set of real numbers such that aRb iff $a - b$ is an integer. Is R an equivalence relation? Justify your answer.

g) Find the particular solution of the recurrence relation $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2$.

h) State Lagrange's theorem. Also discuss the converse of the theorem.

i) Find all the distinct left cosets of $H = 5\mathbb{Z}$ in the group $(\mathbb{Z}, +)$.

j) Explain Boolean algebra with the help of an example.

k) If G is minimally connected then prove that G is a tree.

l) Prove or disprove that there is no connected Eulerian simple graph that has even number of vertices and odd number of edges.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 **a)** Define generalized Pigeon-hole principle. Students are awarded 4 grades A, B, C, and D. How many students must be there in a group so that at least 6 students get the same grade? **(8x2)**

b) Solve the following recurrence relation using generating function

$$a_n - 2a_{n-1} - 15a_{n-2} = 0, \text{ for } n \geq 2 \text{ and } a_0 = 0, a_1 = 1.$$

Q4 **a)** What is a Tautology? Construct the truth table of $(P \rightarrow Q \wedge R) \vee (\neg P \wedge Q)$. **(8x2)**

b) In a distributive lattice, show that if an element has a complement, then this complement is unique.

Q5 Define integral domain. Show that every field is an integral domain but converse is not true. When an integral domain becomes a field? Explain the answer in details. **(16)**

Q6 **a)** Show that a graph G is connected iff it has a spanning tree. **(8x2)**

b) Prove that every plannar graph is 6-vertex colorable.