Registration No : | | | | | | | | | | |

Total Number of Pages : 02

B. Tech./
Integrated Dual Degree (B. Tech & M.Tech)
RMA2A001

2nd Semester Regular/Back Examination: 2022-23
Mathematics II
All branches
Time : 3 Hour
Max Marks : 100
Q. Code : M442

Answer Question No.1 (Part-1) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions : **(2 x 10)**

a) Write the definition of a basis for a vector space V.

b) Under what condition a nonhomogeneous system of m linear equations in n unknowns will have no solution?

c) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ 3 & 1 & 4 \end{pmatrix}$, what are the eigenvalues of the matrix A?

d) Eigen values of skew-symmetric matrices are either______ or ________.

e) Express the straight line parametrically which passes through the point (2, -1, 4) in the direction of the vector (1, 2, -1).

f) Find curl of $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$.

g) State Green's Theorem in a plane.

h) Find the surface normal $\vec{N}$ to the surface f(x, y ,z)= $x^2 + y^2 - z^2$.

i) What is the fundamental period of the function $\cos \pi x$?

j) Define Fourier series of a function f(x) in $(-\pi, \pi)$.

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) **(6 × 8)**

a) Solve the equations 4y +3z = 8; 4x – 2z = 10; 3x +2y = 5 by any suitable method.

b) Find the inverse of the matrix $\begin{pmatrix} 4 & 2 & 1 \\ 3 & 2 & 5 \\ 2 & 0 & 5 \end{pmatrix}$.

c) Show that the product of two orthogonal matrix is orthogonal.

d) Diagonalize the matrix $\begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix}$.

e) Show that the eigenvalues of Hermitian matrix is always real.

f) Find the directional derivative of the function $f(x, y, z) = e^x + e^y + e^z$ at the point P(-4, 2, 3) in the direction $\vec{a} = [1, 2, 1]$

g) Find the area bounded by the line $y = x$ and the curve $y = x^2$.

h) Evaluate $\int_C \vec{F}(\vec{r}).d\vec{r}$ where $\vec{F} = [1, y, z]$ and C: $\vec{r} = [t, \cos t, \sin t]$ from $(0, 1, 0)$ to $(\pi/2, 0, 1)$.

i) Use Stokes' theorem to compute $\oiint_S curl\vec{F}.d\vec{S}$ where $\vec{F} = x^2\hat{i} + 2x\hat{j} + 2z\hat{k}$ and S is the surface given by $x^2 + \frac{y^2}{4} + \frac{z^2}{a^2} = 1, \quad z \geq 0$.

j) Find the Fourier series of the given function $f(x) = \begin{cases} k, & -\pi < x < 0 \\ -k, & 0 < x < \pi \end{cases}$

k) Find the Fourier cosine transform and Fourier sine transform of $f(x) = \begin{cases} 5, & -1 < x < 1 \\ 0, & otherwise \end{cases}$

l) Show by Fourier integral that $\int_0^\infty \frac{\cos wx + w \sin wx}{1 + w^2} dw = \begin{cases} 0, & x < 0 \\ \pi e^{-x}, & x > 0 \end{cases}$

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 Solve the system of equations $2x_1 + x_2 - 2x_3 + 2x_4 = 5$; $4x_1 + 5x_2 - 3x_3 + 6x_4 = 9$; $-2x_1 + 5x_2 - 2x_3 + 6x_4 = 4$; $4x_1 + 11x_2 - 4x_3 + 8x_4 = 2$. **(16)**

Q4 Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$. **(16)**

Q5 Find the Fourier series of the function $f(x) = \begin{cases} 0, & -2 < x < 0 \\ 2x, & 0 < x < 2 \end{cases}$ with period 4. **(16)**

Q6 Evaluate $\oiint_S \vec{F}.\vec{n}\, dA$ where $\vec{F} = [6x, 0, -2z]$, over the sphere S: $x^2 + y^2 + z^2 = 4$ (i) directly and (ii) using Gauss Divergence theorem. **(16)**