**Registration No.:** | | | | | | | | | | |



**Total Number of Pages: 02** **Integrated Dual Degree (B.Tech and M.Tech)**
**23BS1004**

**2nd Semester Regular Examination: 2023-24**
**SUBJECT: Mathematics - II**
**BRANCH(S):**
**AE,AEIE,AERO,AME,AUTO,BIOMED,BIOTECH,C&EE,CE,CHEM,CIVIL,CSE,CSEAI,CSEAIME,CSEDS,CSIT,CST,ECE,EEE,ELECTRICAL,ELECTRICAL&C.E,ETC,IT,MANUTECH,MECH,METTA,MINERAL,MINING, MME,PLASTIC, PT, CE,CSE,ECE,EE,ME**
**Time: 3 Hour**
**Max Marks: 100**
**Q.Code: P220**
**Answer Question No.1 (Part-1) which is compulsory, any eight from Part-II and any two from Part-III.**
**The figures in the right hand margin indicate marks.**

**Part-I**

**Q1** **Answer the following questions:** **(2 x 10)**

**a)** The exact differential equation Mdx + Ndy = 0 will be exact if and only if ________.

**b)** The integrating factor for ydx – xdy = 0 is _______.

**c)** Consider a differential equation dy/dx – y = x with the initial condition y(0) = 0. Using Euler's first order method with a step size of 0.1, find the value of y(0.3).

**d)** What is the differential equation of all parabolas whose directrices are parallel to the x-axis?

**e)** $Curl\ of\ \vec{f}(x,y,z) = 2xy\hat{\imath} + (x^2 + z^2)\hat{\jmath} + 2zy\hat{k}\ is$ __________.

**f)** If C represents a line segment between (0,0,0) and (1, 1, 1) in the Cartesian coordinate system, the value of the line integral given below will be _______.
$\int c\ [(y + z)dx + (x + z)dy + (x + y)dz]$

**g)** Find the value of $\int_0^{\pi} sin^2x\ cos^4x\ dx$

**h)** Express $5^3 = 125$ in logarithm form.

**i)** If f(x) = [x] where [.] denotes greatest integer function and g(x) = 2x then find the value of gof(- 3/2) + gof(5/2) ?

**j)** If $\frac{4+3i}{3-4i} = x + jy$, then what is the value of $\frac{x}{y}$

**Part-II**

**Q2** **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6 × 8)**

**a)** **Solve the following differential equation:**

$$\frac{dy}{dx} + y \sec x = 7$$

**b)** Solve the Bernoulli equation $y' + y = e^x x^3$

**c)** Find a power series solution for the following differential equations.

$y'' + 6y' = 0$

**d)** Solve the differential equation by using a variation of parameters, $y'' + 9y = \sec 3x$

**e)** What is Cauchy Riemann equation for the function $f(z) = \frac{\bar{z}^2}{z}, z \neq 0$ and $f(0) = 0$.
Show that Cauchy Riemann equation are satisfied at (0, 0) but is not differentiable at (0, 0)

**f)** Evaluate the line integral $\int_{AB}(x + y)dx + x\,dy$
1) AB is the line segment from A(0, 0) TO B(1, 1)
2) AB is the parabola $y = x^2$ from A(0, 0) TO B(1, 1)

**g)** Solve for x if $\log(x - 1) + \log(x + 1) = \log_2 1$

**h)** Find the solution of $y'' - 6y' + 13y = 0$, $y(0) = 0$, $y'(0) = 10$.

**i)** State and prove the Residue Theorem.

**j)** The potential that represents an inverse square force is
$V(x, y, z) = K/(X^2 + Y^2 + Z^2)^{1/2}$ where K is a constant. Using the definition
$F = \nabla V$ then calculate the components of this force.

**k)** Evaluate, $\int_c \frac{z^2}{z-5} dz$, where "c" is the circle such that $|z| = 2$?

**l)** Let $V = 4x^2yz^3$ at a given point P (1, 2, 1), then find the potential V at P and also verify whether the potential V satisfies the Laplace equation or not.

## Part-III

**Only Long Answer Type Questions (Answer Any Two out of Four)**

**Q3** **a)** Show that an analytic function with constant modulus is constant. **(8x2)**

**b)** Check f(z) = $z^2$ is analytic or not . Justify your answer

**Q4** **a)** Solve the Legendre's linear equation $[(3x + 2)^2 D^2 + 3(3x + 2)D - 36]y = 3x^2 + 4x + 1$ **(8x2)**

**b)** Solve the initial-value problem.
$y'' + 5y' + 6y = 0$, $y(0) = 0$, $y'(0) = -2$

**Q5** **a)** Find the value of the line integral $\oint_c -2y dx + (3x - 4y^2)dy + (z^2 + 3y)dz$ **(8x2)**

**b)** Calculate the work done on a particle by force field F(x, y) = y + sinx, as the particle traverses circle $x^2 + y^2 = 4$ exactly once in the counter clockwise direction, starting and ending at point (2, 0).

**Q6** **a)** Solve Laplace equation, $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 0$, with the boundary conditions: **(8x2)**
(I) u(x, 0) = 0
(II) u(x, 1) = 0
(III) u(0, y) = F(y)
(IV) u(1, y) = 0 .

**b)** Proof of Cauchy's integral theorem