Registration No : | | | | | | | | | | |

Total Number of Pages : 02

Course: B.Tech
Sub_Code: RNM6A002

6th Semester Regular/Back Examination: 2022-23
SUBJECT: Numerical Methods
BRANCH(S): CSE, CSEAIME
Time : 3 Hour
Max Marks : 100
Q.Code : M137
Answer Question No.1 (Part-1) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q1 **Answer the following questions:** **(2 x 10)**

a) What are the order of convergence of Secant and Newton Raphson method?

b) Explain the geometrical interpretation of Regula Falsi method.

c) Determine L and U to compute the LU factorization of the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 4 & 5 & 2 \\ 2 & -2 & 0 \end{pmatrix}$$

d) Write the sum of 0.123×10^3 and 0.456×10^2 in 3 digits mantissa form.

e) Write the error term in interpolation.

f) Write the Newton-Cotes quadrature integration formula?

g) If $f(x) = \frac{1}{x^2}$, find the divided difference $f[x_1, x_2, x_3]$.

h) How many nodes should be there for Simpson's $\frac{1}{3}rd$ rule?

i) What do you mean by spectral radius?

j) Find $y(0.2)$ by Euler's method, given that $\frac{dy}{dx} = x + y;\ y(0) = 1$ with $h = 0.1$

Part-II

Q2 **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6 × 8)**

a) Find the solution of the below system of equations correct to three decimal places, using the Gauss-Seidel iteration method:

$$45x_1 + 2x_2 + 3x_3 = 58$$
$$-3x_1 + 22x_2 + 2x_3 = 47$$
$$5x_1 + x_2 + 20x_3 = 67$$

b) Find a real root of the equation $\cos x = 3x - 1$ correct to four decimal places by fixed point iteration method.

c) Find the smallest positive real root of the $\tan x + \tanh(x) = 0$ by using Bisection method.

d) Define truncation error. What is the absolute, percentage and relative errors involved if $y = \frac{2}{3}$ is represented in normalized decimal form with 6-digits?

e) Evaluate $\int_{1}^{1.4} e^{-x^2}\,dx$ dividing the range in to 4 equal parts by Simpson's $\frac{1}{3}rd$ rule.

f) Find the missing term in the table using Lagrange's interpolation

x	0	1	2	3	4
y	1	3	9	–	81

g) Evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$ using Trapezoidal rule with h=0.2. Hence obtain an approximate value of π

h) Derive Newton's divided difference interpolating formula which interpolates a function at $x = x_i,\ i = 0,1,\ldots,n$. Mention its error factor.

i) Find the value of $y(0.04)$ using Euler's method, given that $y' + y = 0, y(0) = 1$, $h = 0.01$

j) Find the approximate value of the integral $\int_0^1 \frac{dx}{1+x}$ using composite trapezoidal rule with 2, 3, 5, 9 nodes and Romberg integration.

k) Using improved Euler's method find y at $x = 0.1$ *and* $x = 0.2$, given $y' = y - \frac{2x}{y},\ y(0) = 1$ with h = 0.1.

l) Using fourth order Runge-Kutta method, find $f(1.4)$ if

$$y' = y - x^2 + 1, y(0) = 0.5, h = 0.2.$$

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 Solve the system of equations $Ax = b$, where $A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}, b = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$, using the LU decomposition method. Take all the diagonal elements of L as 1. Also find A^{-1}. **(16)**

Q4 Write three properties of Quadratic Spline interpolation. For the data given below **(16)**

x	0	1	2	3
$f(x)$	1	2	33	244

fit quadratic splines with $f''(0) = M(0) = 0$. Hence, find an estimate of $f(2.5)$.

Q5 Find the general solution of the system of equations **(16)**

$$\frac{dx_1}{dt} = -5x_1 + 2x_2 + t, \frac{dx_2}{dt} = -2x_1 - 2x_2 + e^{-t}.$$

Q6 Find the eigenvalue correct to three decimal places which is nearest to 5 for the matrix $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$ using inverse power method. Obtain the corresponding eigenvector. Take the initial approximate vector as $v^{(0)} = [1,1,1]^T$. **(16)**